HABILITATION THESIS REVIEWER'S REPORT

Masaryk University	
Applicant	RNDr. Lenka Přibylová, Ph.D.
Habilitation thesis	Applied nonlinear dynamics
Reviewer	dr. Hil G.E. Meijer, associate professor
Reviewer's home unit, institution	University of Twente, Department of Applied Mathematics, Netherlands.

[Review text]

The habilitation thesis by dr. Pribylova shows a consistent line of research in nonlinear dynamics and applications. The introduction shows that this research is firmly based on knowledge of the theory. The applicant also shows growth in her methodology as she extends her tools, analytically and numerically over the years. The focus of her research is not the development of these tools but to investigate models for applications and show the possible dynamical repertoire.

In her treatment of models in ecology and population dynamics, she rightfully advocates that one should do nonlinear analysis, e.g. compute Lyapunov coefficients, to predict the stability of bifurcation limit cycles as that may depend on parameter choices. She shows that stable and unstable cycles can appear for different values for the same model. She discusses how hysteresis due to bistability (p127-136) has consequences if we slowly change parameters from one regime to another. I want to highlight that the questions considered in the papers are well-motivated. Every year, many articles appear or are submitted in which the authors consider yet another variant of prey-predator dynamics. In many cases, there is only a tiny variation without great effect or novelty, just changing the functional response without comparison (as is done here). And then, such manuscripts mostly consider linear stability analysis and present some numerical simulations only. A more thorough investigation with two parameters, or an attempt to interpret the results, is often lacking. In this Habilitation, one-parameter diagrams are presented that are useful to interpret dynamics for applications, as well as some two-parameter diagrams with codimension two bifurcation points that act as organising centers. And she does not simply take a single functional response but a parametric family (p75-97) so that her results show how the shape affects the

A full modelling cycle would discuss data from which models are derived, which are then analysed and interpreted. The final step is to come up with predictions that can be tested. It is natural that the applicant starts from existing models and then demonstrates the nonlinear repertoire. The works presented in this habilitation on ecology make reasonable attempts to connect to other fields but still focus mostly on the mathematical analysis of the models. This is apparent on page 55, [Marik&Pribylova 2006, p5]. Here, it is mentioned that lakes exhibiting a predator-only population exist, but no reference or concrete example is given. This also appears on pages 173-174, where she interprets her results for the IS-LM model. There are precise statements about what the results mean, which shows that this is more than lip service to applications. Here I would have liked to see more connections to real data and other literature in that field, but I also know that such papers serve as an invitation for economists and ecologists that are open to modelling to collaborate further.

Pribylova demonstrates such capabilities in her work on Covid19-related modelling, where she works with large international teams, performs data analysis and presents model predictions. Here, Figure A1 on p307 is a big achievement showing how the modifications to the standard SIR model lead to (in hindsight) useful predictions of the epidemic for policymakers.

As a point of criticism, she could improve her command of English, partially for grammar but more for the meaning of the words. Often I could understand what was meant but still would suggest rewriting. This Habilitation is a collection of existing papers, but it could not have hurt to do some additional editorial work.

On p31, the statement “Chaotic dynamics is very sensitive to initial conditions” is not correct. Instead, the common definition of chaotic dynamics characterises chaos as a sensitive dependence (of solutions) on initial conditions. The works on chaotic dynamics (p193-227) are sound, but the novelty of the results is perhaps less important here than the message to the field that such dynamics is present. On the one hand, the works on synchronisation she lists show typical results such as phase-locking, bistability and chaos, but it is important to show and explain these features to engineers and experimentalists, so they understand their data better.

Summarising, I believe the Habilitation shows Lenka Pribylova is an excellent researcher, has worked on topics alone and in collaboration, and is able to supervise. The introduction clearly presents an overarching view of an applied mathematician and divided over many fields rooted in dynamical systems analysis. The broadness of the topics (dynamical systems, ecology, economics, synchronisation, Covid19) with results at the current research frontier is a sign of good talent. More than once, she is correcting publishing results which helps science forward. This habilitation thesis is more than enough for her appointment as (associate) professor.**Reviewer's questions for the habilitation thesis defence**
The questions I list below are there as I would have asked them as a normal reviewer too. They are not dismissing the value of the works, but should be seen as an invitation to make certain details more precise, and to speculate how the results could be extended.

1. The discussion on page 59-60, [Marik&Pribylova, EJDE 2006-p10], suggests that the model in this study has a generalized Hopf bifurcation (Bautin). It is known that for parameter values near the transition from sub to supercritical Hopf, there is a saddle-

node bifurcation of limit cycles. In later papers, this scenario is encountered and analysed but not for this model. The question is first how she would analyse the codimension 2 scenario in more detail, and second how the presence of the Saddle-node of limit cycles bifurcation affects the interpretation of the model.

2. On page 40-41, the applicant says that some numerical aspect in the model is stiff. To me, it is unclear where this stiffness appears. Stiffness leads to small steps in parameters for example as the numerical problem is sensitive. This also appears on page 247, but harmonic terms are not stiff per se. Rather they present a challenge as a proper method has to be formulated that can be solved numerically. Can she explain the numerical problem, state how it is stiff, and how her approach solves this?

3. On page 101, the applicant mentions that limit cycles may be fatal as small perturbations when the state is near the origin could jump to one of the axis such that some subpopulation goes extinct. In these situations, I wonder whether the ODE framework is adequate. Describing such jumps or noise (sudden deaths or immigration from elsewhere) may be better characterized in a stochastic modelling framework, especially because the population size is small. Could the applicant comment on the validity of the Rosenzweig-MacArthur model in case we consider such perturbations?

4. On p120, figure 5 of [Hajnova&Pribylova, 2017], there is a dashed line emanating from the Chenciner point [CH] turning at the Cusp [CP] and terminating at R2. While this is theoretically a correct scenario, finding this line numerically is extremely challenging. The use of MatContM in this chapter and the specification of the numerical values warrant the credility of all local bifurcation curves, and the existence of the global bifurcation curves near the R2 and CH points. Further away this is not clear at all. Can she comment on the numerical methods used to determine the precise shape of the “fold bifurcation of loop”-curve?

   A follow-up question is that this saddle-node bifurcation of invariant curves is a quasi-periodic bifurcation, which implies that the bifurcation set is fractal and very complicated. Is it correct to present this as a “simple” curve?

   A related question is how the applicant sees the reproducibility crises in science? How should we publish our methods?

5. On page 139, figure 1, it is chosen to present the critical case p=2, when the Hopf terminates on the Cusp Point. Could we interpret this as a degeneracy in the normal form, and what are the dynamical implications of this codimension 3 situation?

6. The paper on Josephson junctions discusses dynamics close to a Hopf-Hopf point. The normal form for the double Hopf bifurcation consists of two times two differential equations and one can group them as the first and second Hopf normal form. This is

also done in on page 150, in the paper on the Josephson junction. This standard part is then complemented by a cubic coupling term in both equations so that the amplitude of one Hopf-component affects the other, and they should both be nonzero. In equation (5), this cubic coupling seems to be missing. It certainly is not there in the second oscillator (u,v) with parameter omega, but what about the time scaling with V of the (x,y)-component. Is this a generic Hopf-Hopf point, or not?

7. In applications of the SEIAR model, the novel asymptotic class A is introduced and I believe this is a very interesting addition. An essential component is then to estimate parameters to see which fraction shows symptoms and which part does not. During Covid19 there was extensive sampling and more data. How could this model be applied other corona or influenza-type epidemics when the data is more scarce?

Conclusion

The habilitation thesis entitled “Applied nonlinear dynamics” by Lenka Přibylová **fulfils** the requirements expected of a habilitation thesis in the field of Mathematics – Applied Mathematics.

Date: 14-01-2023 Signature: